AME40453 - Automation and Controls
C6 Pre-Lab Assignment

Please refer to the lab handout for the following questions. Express your answers as algebraic equations written on a separate sheet of paper, and show your work. Then, transcribe the equations into your lab notebook.

1. Consider a pendulum with zero applied torque $\tau = 0$.
   a. Sketch the solution $\theta(t) = e^{-\lambda t}\sin(\omega_d t)$ to Eq. (1) of the handout.
   b. Describe an experimental method to measure λ. (Hint: Think back to the pendulum in Lab I.)
2. Consider a pendulum at rest at some desired angle or "set-point" θ_S. Derive an equation for the applied motor torque τ_S necessary to maintain that angle.
3. Write the function $mgR\sin(\theta)$ as a first order Taylor series expansion about the point $\theta = \theta_S$.
4. Write an approximate version Eq. (1) of the handout using the first order Taylor series you just derived.
5. Express the equation of motion as a linear system of first order differential equations using the new variable substitution $\dot{\theta} = \omega$ and $\ddot{\theta} = \dot{\omega}$.
6. Tie it all together now: Rewrite the linear system of differential equations in LQR form $\dot{x} = Ax + Bu$ where $x = \begin{bmatrix} \theta - \theta_S \\ \omega \end{bmatrix}$ and $u = \tau - \tau_S$. In particular, what are A and B in terms of m, g, R, and γ?

7. Develop an LQR controller for a pendulum with $R = 0.15$m, $m = 0.05$ kg, $\lambda = 0.7$ s^{-1}, and a set point $\theta_S = 45°$.
   a. The motor we will use in lab has a limit on how much torque it can output. Look at the torque-speed curve for the motor in Appendix C. What is the maximum torque output τ_{max}?
   b. Write down expressions for the LQR weights **Q** and **R** in terms of maximum angular displacement $\Delta\theta_{max}$, maximum angular speed ω_{max}, and maximum motor torque τ_{max}.
   c. Use the lqr() method in Matlab to calculate the gains k_p and k_d (units of Nm/rad and Nms/rad, respectively) for a setpoint $\theta_s = \pi/4 = 45°$, a maximum angular displacement $\Delta\theta_{max} = 0.087\ rad = 5°$, and a maximum angular speed $\omega_{max} = 0.2$ rad/s. Save the script in your code library. You will need it in lab.