

AME40453 - Automation and Controls  
C4 Pre-Lab Assignment

**For the following questions, please express your answers as algebraic equations written on a separate sheet of paper, and show your work. Then, transcribe the important equations into your lab notebook.**

1. Use a conservation law (rate balance for the volume of water) to derive a differential equation for the height of the water in the tank. Use Poiseuille's Law  $p = RQ_{OUT}$  and the hydrostatic pressure  $p = \rho gh$  to determine the flow rate out. Assume the flow rate in  $Q_{IN} = S$ , where  $S$  is some constant flow rate provided by the pump.
2. Derive an equation for the time constant  $\tau$  in terms of  $R$ ,  $\rho$ ,  $g$ , and the cross sectional area of the tank  $A_T$ .
3. Consider the "quiescent" mode where the tank is at steady state (i.e. the fluid height is not changing). Derive an equation for the flow rate  $S_s$  that yields a desired equilibrium height  $h_s$  in terms of *only* the variables  $A_T$ ,  $\tau$ , and  $h_s$ .
4. Use the definition of the LQR variables  $x = h - h_s$ ,  $u = S - S_s$  to show that  $u = -k_p x$  and Eq. (3) are equivalent expressions.
5. Using your previous answers, show that Eq. (4) is equivalent to the governing differential equation from problem 1, if  $x = h - h_s$ ,  $u = S - S_s$ ,  $A = -1/\tau$ , and  $B = 1/A_T$ .
6. Use the `lqr()` method in Matlab to calculate the optimal gain  $k_p$  (in units of  $\text{in}^2/\text{s}$ ) for a tank with a diameter  $D = 5$  in. and height  $h_{max} = 1$  in. The pump has a max flow rate  $S_{max} = 15 \text{ in}^3/\text{s}$ . Draining the tank has a characteristic time constant  $\tau = 6\text{s}$ . Use  $\mathbf{Q} = 1/|h_{max}|^2$  and  $\mathbf{R} = 1/|S_{max}|^2$  for the LQR weights.