AME 20216 - Lab I
Technical Memo
Date Submitted: ** fill in correct date here **
Dates Performed: ** fill in correct date here **
To: Prof. Ott and Dr. Rumbach
From: ** your name or NDID# here **
Subject: ** fill in the title of lab exercise here **

Summary:

In this work, two types of studies were conducted. The first was the calibration of a large-scale strain gage and the second was the use of cantilever beam to measure the mass of a penny and an unknown material (a cylinder), which was then identified by its density. For the calibration study, the large-scale strain gage showed a linear relationship with a high coefficient of determination (r^2 = 0.999) and a small relative standard error that was far less than 1% at 95% confidence. It was also determined that the cantilever beam apparatus in full bridge mode was more sensitive with narrower confidence intervals than in the quarter bridge configuration. The measured mass of the penny for the two configurations was 4.08 g and 4.10 g. respectively, both greater than the published value of 2.500 g by more than 70% This was attributed to dirt and other contaminants on the pennies skewing the data. The mass for the cylinder was measured to be 305.8 g (full bridge) and the volume was 35.941 cm^3, such that the density was 8.501 g/cm^3 in the full bridge configuration, and 288.7 g and 8.058 g/cm^3 in the quarter bridge configuration. In comparison with published data of common materials, the unknown material was identified to most likely be brass or bronze.

Findings:

In this work, two studies were conducted: 1) a ‘macro’ strain gage was built and calibrated, and 2) strain gages in two different Wheatstone bridge configurations (full and quarter) on a cantilever beam were used to determine the mass of both a single penny and the density of an unknown object (a cylinder). For both studies, calibration was required and linear regression analysis was implemented utilizing the least squares approach, and quantitative conclusions were drawn based on the data.

Study 1: The large-scale strain gage was constructed from a stretching apparatus, dial, gauge, stainless steel wire, and a multimeter. The wire was stretched known amounts and the resistance was measured across the entire length of the wire. The resulting measured resistance was related to the strain, defined as $\varepsilon = \Delta L/L$ where L is the initial wire length and ΔL is the known change in wire length, and a calibration curve was fit to the resulting resistance data using the unweighted least squares method. Recalling from [1] that for linearly related properties the calibration curve will take the following form

$$R = a_0 + a_1\varepsilon, \tag{1}$$

the unknown coefficients can be determined by

$$a_1 = \frac{\overline{\varepsilon R} - (\overline{\varepsilon})(\overline{R})}{\overline{(\varepsilon^2)} - (\overline{\varepsilon})^2} \tag{2}$$

$$a_0 = \overline{R} - a_1\overline{\varepsilon}, \tag{3}$$

where the overbar indicates a mean quantity. Details of this calculation are included in Appendix A. The resulting data, calibration curve, and confidence (precision) interval are plotted in Fig. 1, where

$$R = 211.96 + 515.64\varepsilon \qquad (\Omega), \tag{4}$$

Several remarks can be made about the sensitivity of the calibration curve, quality of the curve fit, and the range to which it applies. Table 1 contains useful statistical parameters pertaining to the data. The slope of the regression line K is referred to as the sensitivity of the calibration curve and takes of a value of K = 515.65 Ω. The slope is positive but of $\mathcal{O}(10^0)$ suggesting relatively

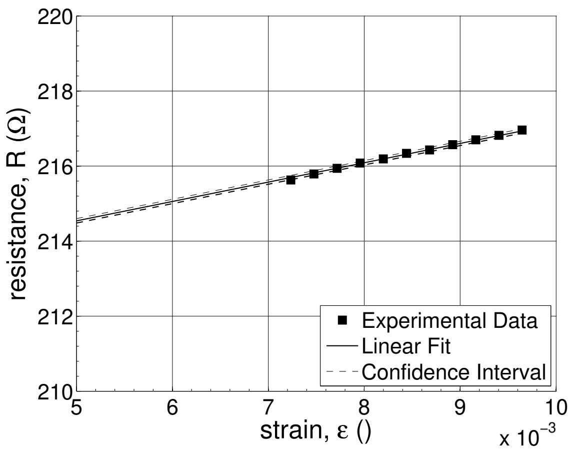


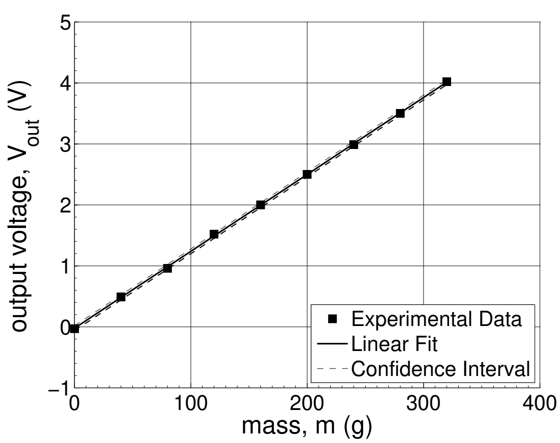
Figure 1: Calibration curve relating the resistance R to the known strain ε for the 'macro' strain gage.

low sensitivity. The standard error $t_{10,95\%}S_{yx}$ was 5.7928×10^{-2} at 95% confidence where the Student's t value was $t_{10,95\%} = 2.228$ [2]. Given that the measure resistance is $\sim 200\ \Omega$, the relative standard error is less than 2.6811×10^{-2} %, suggesting that the curve fit was good. Similarly, the coefficient of determination r^2 is asymptotically close to 1 implying a near perfect correlation, giving high confidence that the relationship is in fact linear. Because the strain gage was only calibrated across a strain of $\varepsilon = 0.3 - 0.4$, this is the suitable range of use for this system. Due to the relative sparseness of data below this range and the prospect of possible mechanical failure above it, this system is not recommended for use outside this range without both acquiring more data or mechanically fortifying the system. Details of the calculations used to obtain the values in Table 1 can be found in Appendix A.

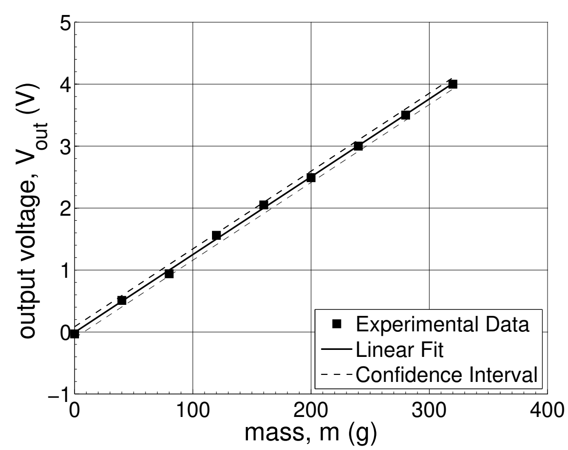
Table 1: Curve fit parameters for the large-scale strain gage. The degrees of freedom was $\nu = 7$ and R_{ci} represents calculated values from Eq. (3) while R_i represent raw resistance data.

Parameter	Value
sensitivity, K	515.65 Ω
standard error, S_{yx}	5.7928×10^{-2} Ω
coefficient of determination, r^2	0.99964

Study 2: For the second study, a cantilever beam was outfitted with four strain gages. In a Wheatstone bridge configuration. All four strain gages could be utilized simultaneously in a full bridge configuration or three of them could be replaced by fixed resistors ($R_{fix} = 120\Omega$) in a quarter bridge configuration. Weight could be applied to the end of the cantilever beam and the resulting deflection sensed by the strain gage measurement system, thus it could be used as a scale. A calibration curve was generated for each configuration of the cantilever beam apparatus relating the measured mass and output voltage from the measurement system. For a known mass m, voltage V_{out}out was measured over the Wheatstone bridge circuit and the corresponding calibration curves along with precision intervals are shown in Fig. 2. The calibration curve for the full bridge configuration yielded a smaller precision interval indicating that the full bridge produced more consistently linear data and a higher quality curve fit. It is worth comparing the slope of the calibration curves for each configuration. For the full bridge $K_{FB} = 0.012613$ while for the quarter bridge $K_{QB} = 0.012558$, which are less than 1% different indicating that the full bridge configuration was negligibly more sensitive.



(a)



(b)

Figure 2: Calibration curve relating the output voltage Vout to the known mass m for (a) the full bridge configuration and (b) the quarter bridge configuration.

This apparatus was used to measure the mass of a single penny as well as several discrete quantities of pennies. The mass of a single penny is 2.500 g according to the United States mint [3], and the measured results were compared graphically to the predicted mass $m = 2.5n$ (g) in Figure 3, where n is the number of pennies. A simple qualitative assessment of the data shows a decline in accuracy as the measured number of pennies increases as both configurations over predict the theoretical mass. This is can likely be attributed at least in part to the aggregate amount of dirt and other contaminants increasing as the number of pennies increases. For the two measurement configurations there is a slight discrepancy in the measured value of single penny; however, it would seem that as the mass increases the two techniques yield results that steadily approach each other. Table 2 lists the masses measured for the two configurations and the percent difference relative to the theoretical value. For this measurement the quarter bridge method yielded data closer to the published value, but this is possibly due to a less sensitive calibration curve (as shown in Fig. 2b) as opposed to an inherently more accurate system.

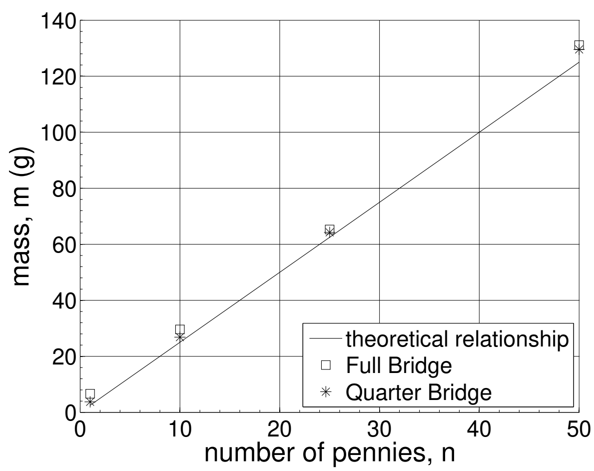


Figure 3: Measured mass m as a function of the number of pennies n for the full and quarter bridge configuration along with the predicted value from [3].

The cantilever beam apparatus was also used to measure the mass of an object of unknown material, in this case a cylinder. The volume V of the material was determined based on measurements of the length L and diameter d using calipers and the relationship $V = (1/4)\pi d^2 L$. The ensuing density ρ was than calculated as

$$\rho = m/V \tag{5}$$

Table 2: A comparison of the measured and predicted values for the two measurement configurations.

number of pennies	predicted mass (g)	measured mass full bridge (g)	% difference	measured mass quarter bridge (g)	% difference
1	2.500	6.6252	265.0	3.751	150.0
10	25.00	29.62	11.85	26.84	10.74
25	62.50	65.30	4.180	64.27	4.110
50	125.0	131.1	2.100	129.6	2.070

Table 3 outlines the measured values using the full and quarter bridge configurations. Similar to the penny measurements, the quarter bridge configuration measured a mass lower than the full bridge, resulting in an approximately 5% difference in calculated density. Based on these measured values, there are a wide range of candidate materials based on a density between 8.0 - 8.5 g/cm^3 for the cylinder according to [4], including brass alloys, bronze alloys, cadmium, cobalt alloys, and stainless steels. Based on the color and likely cost, many of these can be disregarded and a reasonable judgment is that the cylinder is made from a brass or bronze alloy, It is important to note that this is just speculation, and without knowing more material properties a definitive material classification is not possible. Again, detailed calculations are included in Appendix A.

Table 3: The calculated density for the cylinder of unknown material.

volume (cm^3)	m_{FB} (g)	ρ_{FB}(g/cm^3)	m_{QB} (g)	ρ_{QB}(g/cm^3)
35.940	305.8	8.501	288.7	8.058

Conclusions:

A large-scale strain gage was constructed and calibrated. While the error associated with this system is minimal, its range of operation is limited. Strain gages in Wheatstone bridge configuration fixed to a cantilever beam created a mass measurement system that was calibrated and utilized to measure the mass of a penny and an unidentified material. The system generally over predicted the theoretical mass of the penny. Finally based on mass and volume measurements, the density of a cylinder of unknown material was measured and identified to likely be brass or bronze.

References:

[1] AME20213, Fall 2011, ”Measurement Systems/Calibration”, Lab Exercise #1 Week A, University of Notre Dame, Notre Dame, IN.
[2] Dunn, P. F., 2009, Measurement and Data Analysis, University of Notre Dame, Notre Dame, IN, Chap. 6.9.
[3] The United States Mint, 2011, ”Coin Specifications”, http://www.usmint.gov/about_the_mint/?action=coin_specifications.
[4] ThermTest Inc., 2011, ”Material Property Search Results”, http://www.thermtest.com/material-property-search/.