AME 20216 - Lab I
Technical Memo
Date Submitted: April 14th, 2022
Dates Performed: April 5th, 2022
To: Prof. Ott and Dr. Rumbach
From: [redacted]
Subject: Experiment A11 - Transient Signals

In the first part of the experiment, a thermocouple was used to explore the first-order transient response of heat transfer. First, the thermocouple was connected to the LabQuest unit used to collect and store the temperature data. After the thermocouple was tested, one Styrofoam cup was filled with warm water from the lab sink and one was filled with ice water. The thermocouple was then set in the warm water until the temperature stabilized. Subsequently, a 10-second data collection period was started and the thermocouple was quickly transferred to the cold water. This represented the cooling process of the thermocouple. A heating trial was also conducted in which the thermocouple started in the cold water and was transferred to the warm water. The recorded transient temperature as a function of time can be seen in Figure 1, below.

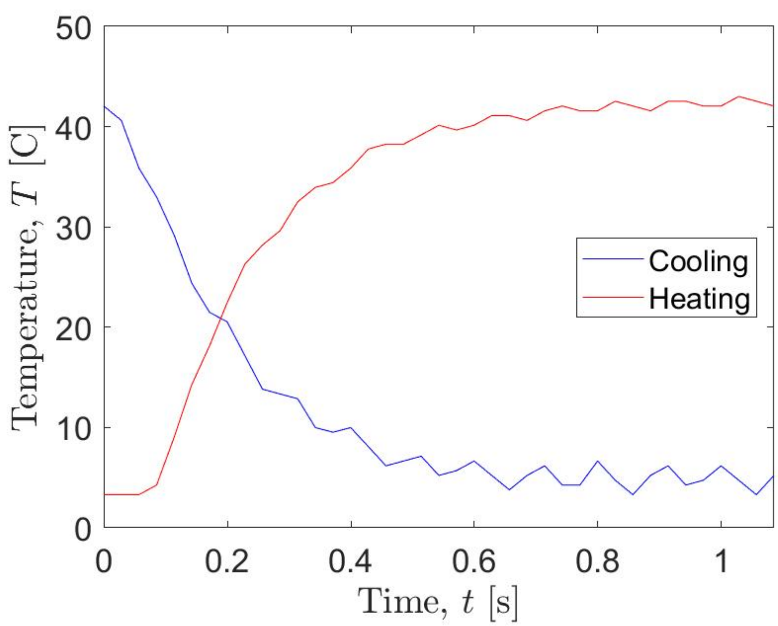

Figure 1: The transient temperature, T, plotted as a function of time, t.

In order to find the time constants for the heating a cooling processes, the transient temperature data had to be linearized. Following a derivation from an explicit solution from an energy balance equation, the data was linearized using the equation

$$y(t) = \ln\left(\frac{T(t) - T_\infty}{T_0 - T_\infty}\right) \tag{1}$$

where $y(t)$ was the transformed data, $T(t)$ was temperature as a function of time, T_∞ was the steady-state temperature of the surrounding fluid, and T_0 was the initial temperature of the surrounding fluid. The data was then cropped to only include the linear portion of the data which reflects the exponential portion of the original data. Linear curve fits were then applied to both cropped portions of data and plotted on the same plot as seen in Figure 2, below.

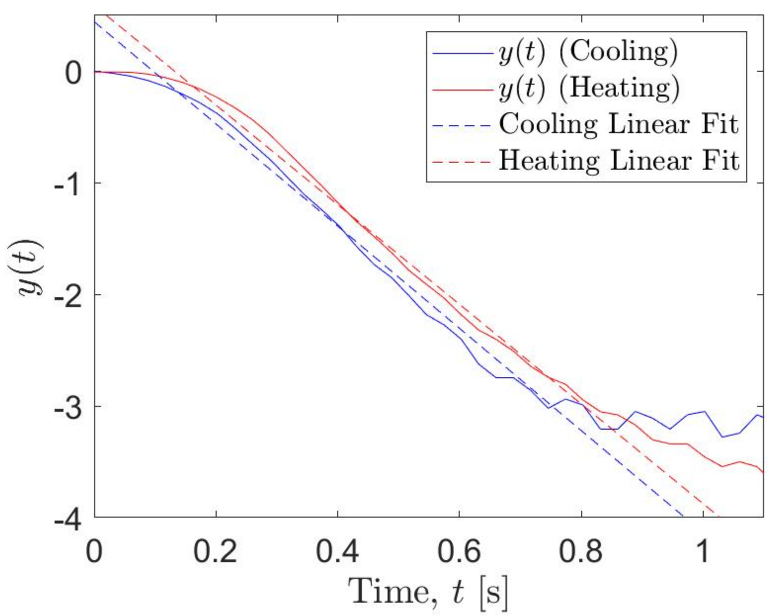

Figure 2: The linearized temperature data, $y(t)$, plotted as a function of time, t. The dashed lines represent linear fits for their respective data sets when the temperature was changing exponentially.

From the linear lines of best fit, the time constant data could be extracted from the slope as the equation for the lines of best fit was

$$y(t) = \left(\frac{-1}{\tau}\right)t \tag{2}$$

where $y(t)$ was the transformed data, τ was the time constant, and t was time. The time constant for heating was $\tau_H = 0.2239$ s and $\tau_C = 0.2183$ s for cooling as also seen in Table 1, below. These values were very similar which aligns with what one would expect. The time constant is roughly the time is takes for the thermocouple to get 63% of the way to the steady-state temperature. Given that the absolute value of the difference in temperatures for both the heating and cooling processes were very similar, I would expect the thermocouple to take a similar amount of time to heat and cool off given that all of its attributes remain the same. Therefore, it makes sense that the time constant values are similar.

In the second part of the lab, a bat was used to explore the attributes of a damped harmonic oscillator. Since the bat experiences some elastic deformation when the ball impacts the barrel and it has mass, it vibrates and is disturbed from equilibrium. This means it can be modeled as a harmonic oscillator per source [1]. However, the fact that a person holds the bat in their hand results in a dampening of the vibrations. Given this additional characteristic, the bat is a good model of a damped harmonic oscillator. While both single and double walled bats were used in the experiment, this specific data that was analyzed was only from the double-walled bat.

Four strain gauges were attached to the barrel of the bat to measure the vibrations. These strain gauges were connected to an amplifier that ultimately outputted the data to an oscilloscope from which the data was saved. While the data was collected as a voltage, this directly related to the displacement of the bat. The CSV file was cleaned of its headers so the data could be loaded into the provided fast Fourier transform (FTT) code. Once this code was executed, the spectral density vs. frequency plot was used to identify the ringing frequency, ω_d. The ringing frequency was identified as the frequency at which spectral density peaked which was determined to be 154.8 Hz. The relevant plot can be seen in Figure 3, below.

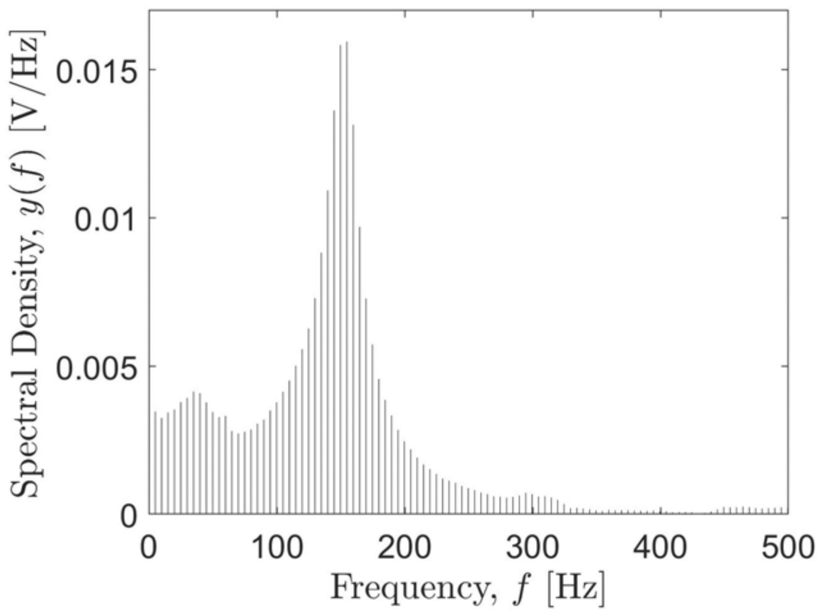

Figure 3: The spectral density, $y(f)$, plotted as a function of frequency, f.

Once the ringing frequency was determined, the value was used in the phase vs. frequency plot produced by the FTT code to find the phase (in rad). The decay constant, λ, was solved for using equation

$$\lambda = \frac{\ln\left(\frac{y_1}{y_2}\right)}{\Delta t} \tag{3}$$

where λ was the decay constant, y_1 and y_2 were two peak values from the strain gauge output vs time graph, and Δt was the time step between the y_1 and y_2 values. Once the decay constant was calculated, the theoretical ringing behavior was predicted using the equation

$$y_{bat}(t) = Ae^{-\lambda t}\sin\left(\omega_d t + \phi\right) \tag{4}$$

where $y_{bat}(t)$ was the strain gauge output, ω_d was the ringing frequency, λ was the decay constant, A was the relative constant amplitude, and ϕ was the phase. Eq. 4 was used to create the theoretical curve in Figure 4, below. The actual strain gauge output is also plotted in Figure 4.

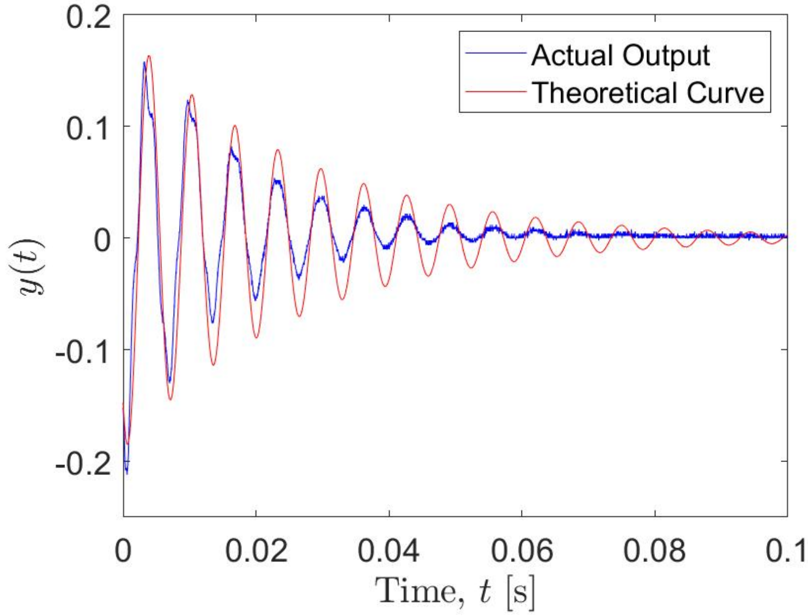

Figure 4: The strain gauge output from the bat, $y(t)$, plotted as a function of time, t. The theoretical curve for the strain was also calculated and plotted.

Finally, the dampening ratio was calculated using the equation

$$\xi_B = \frac{\lambda^2}{\lambda^2 + \omega_d^2} \tag{5}$$

where ξ_B was the dampening ratio, ω_d was the ringing frequency, and λ was the decay constant. Since 4 exhibits an curve indicative of an under-damped oscillator per source [2], the dampening ratio was expected to be less than 1. As can be seen in Table 1 below, this is the case. Table 1 also includes important values from the rest of the experiment.

Table 1: The values relevant to the analysis of the First-Order Transient Response and dampened harmonic oscillator (double-walled bat).

τ_H (s)	τ_C (s)	Ringing Frequency, f_d (Hz)	Damping Ratio, ξ_B
0.2239	0.2183	154.8	0.001476

Overall, the theoretical and actual output curves are relatively similar which suggests the data collection and data analysis was largely successful. However, some errors including the slow sampling rate of the thermocouple and the inconsistency of bat swings could be resolved to create better results.

References:

[1] Russell, D., 2011, ”The Simple Harmonic Oscillator”, The Pennsylvania State University, https://www.acs.psu.edu/drussell/demos/sho/mass.html.

[2] Rumbach, P., 2021, Undergraduate Lectures on Measurements and Data Analysis, University of Notre Dame, Notre Dame, IN, Chap. 22.2.